Uncoordinated MAC for Adaptive Multi-Beam Directional Networks: Analysis and Evaluation

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Abstract—**In this work, we analyze the capacity of airborne networks where each node is equipped with digital multi-beamforming antennas. With this technology, a node can form multiple simultaneous transmit or receive beams per aperture under the constraint that no aperture can simultaneously transmit and receive. Given the potentially large distances between aircraft, propagation delays are significant relative to transmission times, hence traditional CSMA approaches are not appropriate. We first present our model of these multi-beamforming capabilities and the resulting wireless interference. We then derive an upper bound on multi-access performance for an idealized version of this physical layer. We then present a new Distributed MAC for Multi-beam Systems (DM^{2}S) scheme and show that this random access protocol achieves the performance upper bound, albeit at the cost of added delay. We also consider the impact of numerous practical considerations including stochastic arrivals, latency, and power constraints on the performance of our random access MAC protocol. Finally, we present a system implementation and evaluation approach that demonstrates the feasibility of the physical layer technology as well as the DM^{2}S scheme.**

I. Introduction

Today, most widely deployed airborne communications waveforms are omni-directional and cannot support the desired data rates, number of nodes, or the necessary communications ranges to support emerging applications and missions. Furthermore, the capacity of omni-directional waveforms is limited by the high amount of multi-user interference. Directional systems hold the promise of higher capacities, increased scalability, and improved robustness, however, many directional network implementations fail to reach the full potential afforded by directional systems due to their use of simplistic networking and channel sharing schemes. With current system designs, the need for accurate pointing requires a high level of coordination. To achieve the highest link data rates, both the transmitting node and receiving node must simultaneously form a beam towards each other. The difficulty in determining and distributing a schedule to coordinate these transmissions and receptions in a mobile ad-hoc network has in practice led to very constrained topologies.

As mentioned, one approach for system design is to determine and disseminate a coordination schedule for node transmission and reception in a distributed manner. Scheduling in networks of mobile nodes is difficult due to the dynamic changes in link states (i.e., delay, interference, mobility), traffic patterns and the overhead required in schedule dissemination. Furthermore, in networks with large propagation delays, such as airborne networks, handshaking based random access protocols such as carrier sense multiple access (CSMA), typically used in ground networks, are ineffective as propagation delays can be significantly greater than the packet transmission time. In an airborne network, node separations can range from 10 - 500km, hence propagation delays can range from 33μs to 2 ms. We expect link data rates of up to 1 Gbps and hence, the transmission time of 1Kb packet is on the order of 1μs.

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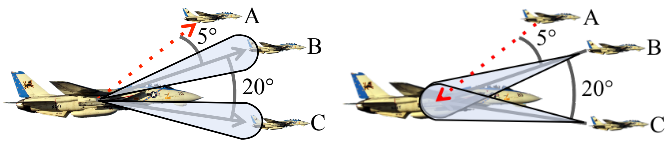
Rather than carefully coordinating the pointing of a single transmit and receive beam between pairs of nodes, we propose to use multiple apertures consisting of multi-element digital antenna arrays and digital beamforming technologies to form a cluster of receive beams that is pointing in all directions simultaneously, thus providing a continuous 360 degrees field of view coverage for each receiving node. This enables the use of low-complexity channel access techniques previously not feasible for directional networks, thereby greatly increasing the network topology flexibility while reducing coordination overhead and hence providing a practical way of achieving the significant capacity gains possible with directional networking. We also consider employing multiple transmit beams to provide further gains in network capacity. These multiple transmit beams can be achieved via multiple array apertures with one or multiple transmit beams per aperture. Multi-element antennas and digital signal processing technology have advanced to the level where multiple transmit and receive beams can be formed extremely rapidly, within microseconds. The requisite neighbor discovery and asynchronous multiple packet acquisition algorithms that take advantage of these capabilities (rapid switching, multiple receive beams, multiple transmit beams) are also being developed. In this paper we develop topology management and channel access technologies that leverage digital arrays and modern signal processing to form a directional mesh networking system that provides higher throughputs and network capacity.

Digital beamforming utilizes an array of omnidirectional antennas each equipped with and Analog-to-Digital converter and with digital signal processing to receive and transmit in an adaptive, spatially sensitive manner. A digital beam is formed



(a) Multi Transmit (b) Multi Receive

Fig. 1: Example of Digital Multi-Beamforming (DMB): (a) multiple simultaneous transmit and (b) multiple simultaneous receive beamforming with a minimum spatial separation $\theta = 10$ degrees. The center node cannot form a transmit beam or receive beam toward neighbor A simultaneously with neighbor B as their angle of separation is within θ degrees of each other.

on the receiving end by applying weights to the received signal vector at each of the elements in the antenna array to generate constructive interference in an adaptive and spatially sensitive manner [1]. Multiple receive beams in different directions can be formed simultaneously. We assume the receiver has the associated packet acquisition capabilities to receive and capture multiple packets from multiple directions simultaneously. To transmit, a complementary process is used to form a beam in the direction of the destination [2], [3]. There are two mechanisms that enable multiple simultaneous transmit beams. Some aircraft have multiple apertures, each with large multi-element arrays. Each aperture is assumed to be able to independently form a single transmit beam or form multiple receive beams. Alternatively, with additional signal processing, a single aperture can be used to simultaneously form beams to multiple receive nodes. Each beam potentially carries independent information. The total transmit power is fixed, hence forming multiple transmit beams results in splitting the power between the multiple beams. Given these assumptions for the physical layer beamforming capabilities, we develop and assess technologies for sharing the wireless channel. We begin by calculating and upper bound on the capacity of a network with these multiple beam capabilities. We then design a random access protocol and show that this protocol achieves network capacity.

II. Related Work

There have been numerous studies on the throughput of single beam directional networks [1], [4]–[7]. However, many of these studies are protocol-based and evaluated via discrete event simulators; little analytical work has been done for single beam directional networks [8]–[10].

There are also works which analyze the throughput of networks with multi-packet reception (MPR) [11], [12] using 802.11 WiFi [13], Code Division Multiple Access (CDMA) [14], Multiple-Input Multiple-Output (MIMO)[1] [16]. However, they do not consider multi-packet transmission as well.

Finally, existing studies which consider multi-transmit and multi-receive beamforming capabilities are access-point based and utilize carrier-sense multiple access (CSMA) or RTS/CTS-based schemes [3], [17]–[19] or require coordination (i.e., two-hop neighbors) [20]–[22].

A review of adaptive antenna array techniques which can perform multi-packet transmission or reception can be found in [2]. However, following a common assumption, the beamforming techniques are assumed to be too complex for mobile nodes and are reserved to the basestation. Unlike previous work, in this work, we consider networks with (1) large propagation delays between nodes making RTS/CTS schemes ineffective and (2) homogenous devices with multi-beam capabilities as outlined in Section III-A.

[1]Unlike beamforming, MIMO utilizes multiple antennas to form numerous low data rate signals. Additionally, MIMO works focus on interference cancellation (e.g., [15]), and are out of the scope of this work.

III. Model and Problem Definition

In this section we outline the network model, assumptions, and performance metrics as well as the throughput problem definition.

A. Beamforming Capabilities

In this work, each node is equipped with a digital beamforming antenna. There are numerous benefits to using beamforming antennas (e.g., improved transmission reliability and transmission range extension [1]). However, in this work, we focus on analyzing the capacity improvements stemming from the increased spatial reuse of forming multiple beams. As such we will leverage the following capabilities:

Observation 1: [Multi-beam transmit, Fig. 1(a)] A node can transmit up to $B_{\max}$ simultaneous directional beams of width θ degrees to their neighbors.

As described in the section above, a node can radiate power in multiple directions, forming distinct simulatenous transmissions. However, this process is limited by the angle of the radiated power, termed the *beamwidth*, and denoted by θ. As such, a node cannot form two transmissions within θ degrees of one another. Additionally, this process is limited by the processing and power constraint of the node. Hence, each node is limited to forming $B_{\max}$ simultaneous transmission beams.

Observation 2: [Multi-beam receive, Fig. 1(b)] A node can receive simultaneous directional beams, provided they are separated by at least θ degrees.

On the receiving node, a transmission can only be discerned from that of a neighbor if they are separated by θ degrees. In the case of two or more beams arriving within θ degrees, we say that none of the beams can be correctly decoded. It is important to note that, using digital signal processing techniques to constantly sample and decode the antenna array, this allows digital beams to be formed on the receiving end of the antenna *after* a packet has already arrived at a node. This effectively enables a digital beamforming antenna to act in an omnidirectional mode, but have antenna gains equivalent to directional mode.

Observation 3: [No transmit while receive] A node cannot simultaneously form a transmit beam while forming a receive beam.

This third and final observation stems from the fact that a transmitting beam will interfere with any receiving beam in the

antenna antenna array. We will assume that anytime a message arrives while a node is transmitting a beam, that message is lost. However, it should be noted that the message which is transmitted can still be correctly decoded at it's destination.

B. Network Model

There are n mobile airborne nodes (assumed to be on a 2 dimensional plane) and all nodes are within range of each other. However, as beamforming nodes can be quite far from one another and still be within communication range[2], there can be significant propagation delays.

We assume that all nodes have pointing, acquisition, and tracking knowledge of each others whereabouts such that they can determine the angle of a beam to a neighbor. Due to the distance between the nodes and a small transmit angle (e.g., $10°$), even relative information of a neighbors location is easy to point and track.

Each node is equipped with a DMB antenna (see Section III-A). We will assume a packet network, with fixed unit length transmission times. Nodes can have two types of traffic: backlogged or stochastic arrivals. Initially, to analyze the capacity of the network, we will first consider the case where all nodes have an infinite buffer of packets to send to each of their neighbors. In Section V-A, we will relax this assumption and consider nodes which have packet arrivals according to a poisson process with arrival rate λ.

C. Problem Definition

A node wishes to maximize the traffic being sent to each of their $n-1$ neighbors (all-to-all traffic). As a performance metric, we will consider the *per-link throughput*.

Definition 1: For a directed link between a source and a destination, the **per-link throughput** is the percentage of time that the source is transmitting and the packet is successfully received at the destination.

Note that as this is a fully connected graph, there are n^2-n directed links in the network.

Fundamentally, this problem is made difficult by the lack of coordination causing each node to act independently. The throughput is composed of the rate that a node transmits (packets/second) and multiplied by the likelihood that the packet is successfully received at the destination. To be successfully received, the destination node must not be transmitting any packets for the duration of the packet arrival. As the packet will arrive at the destination after some unknown (and potentially long) propagation delay, it is very difficult to have the destination node anticipate a packet arrival. Therefore, by having a node transmit often, it will result in lower success rates. Hence, there is a tradeoff for attempting to transmit often with numerous collisions or transmitting infrequently with fewer collisions. In the next section, we will demonstrate an *uncoordinated* stochastic protocol which can achieve the optimal network throughput, even if it were coordinated.

[2]The range of airborne beamforming nodes can be up to 500km, as all nodes have Line-of-sight (LOS) to each other.

TABLE I: Nomenclature

n	Number of nodes within communication range of one another
θ	Beamwidth size (degrees)
B_{max}	Maximum number of transmit beams
T	The per-link throughput

IV. Throughput Analysis

We now begin the throughput capacity analysis for a network of n nodes with DMB antennas whose capabilities were outlined in Section III-A. To understand the upperbounds on throughput capacity, in this section we consider an idealized physical layer with no beamwidth or power constraints.

In the analysis below, we first derive an upperbound on the capacity of such a system. Following this, we show that a random access MAC can also achieve the same capacity as the scheduled MAC in the asymptotic limit of delay.

A. Throughput Upperbound

As an upperbound, we will consider the throughput of the muti-beam sectorized directional system under relaxed versions of Observations (1) and (2) that nodes can form any number of beams ($B_{\text{max}} = n-1$) of arbitrarily small angle ($\theta = 0$). Therefore, the main constraint stems from Observation (3) that a node cannot simultaneously transmit and receive a packet. To derive the upperbound, we will assume that the nodes are perfectly synchronized and have aligned slots.

Intuitively, when considering scheduling in such a system, the problem reduces to a graph vertex coloring. Put simply, if there exists a vertex coloring with only 2 colors (i.e., the graph is bi-partite), then the optimal scheduling is to schedule all of the black nodes in a single slot to transmit to all of their neighbors, each of which are red nodes. In the subsequent slot, schedule all of the red nodes. In this scenario, clearly each link will be activated every other slot, and all packets will be successfully received. Hence, the throughput per link is 50% and this is an upperbound. However, it is clear that such an upperbound will not be able to be achieved in practical scenarios where network topology is not a bi-partite graph.

For a complete graph (as described in Section III), we can also determine an upperbound on throughput.

Theorem 1: The upperbound on per-link throughput is,

$$T^* = 25\%. \tag{1}$$

Proof: With nodes synchronized, slotted, and no propagation delay, the throughput in a slot is completely determined by the set of nodes which are transmitting vs. the set of nodes which are receiving. As a simple example, if one node transmits in a slot to the remaining $n-1$ nodes, then there are $n-1$ successfully activated links in that slot. Following this, if j nodes transmit in a slot to $n-j$ nodes which receive, the number of successfully activated links will be $j*(n-j)$. It is easy to see that an upperbound on throughput will then be when $n/2$ nodes transmit to $n/2$ nodes for an aggregate of

Distributed MAC for Multibeam Systems (DM^{2}S)

Input: Beamwidth θ, Power constraint B_{max}, Node i, Angles α_j to all neighbors j

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while Queue of waiting packets is not empty do
    S = ∅
    for each neighbor j do
        if |α_i − α_j| > θ ∀i ∈ S and |S| ≤ B_max then S = S ∪ i
    Form a transmit beam to all neighbors in S.
    Enter into receive mode for a random period of time R
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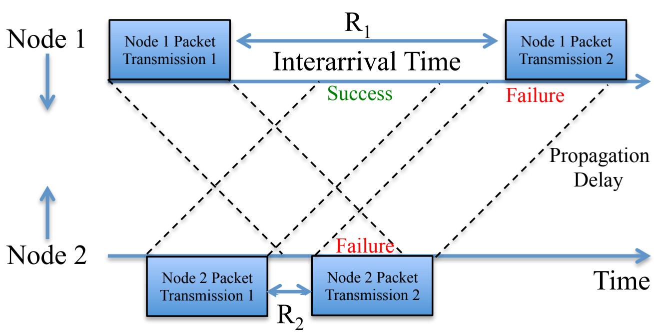


Fig. 2: Random Access MAC Overview: After waiting a random period of time, R, a node transmits a unit-length packet on it's link. If the entire packet arrives within the stochastic interarrival time R, it is successfully received.

$n^2/4$ activated links. As there are $\approx n^2$ links, this results in a per link throughput upperbound of 25%.[3] ∎

Again, however, this scheme requires that nodes are perfectly synchronized (i.e., aligned slots) and coordinated which is difficult, if not impossible, to achieve in practice. Nonetheless, in the following section, we will propose and analyze a number of schemes which will approach this upperbound of 25% throughput per link.

In the section above, we demonstrated that 25% activation per link is an upperbound on the throughput. In the networks of interest, a scheduled system does not work as coordination is too difficult to achieve in an ad hoc network. This is amplified in airborne networks where propagation delays are non-negligible. Furthermore, carrier sense multiple access (CSMA) is not practical as propagation delays cause inaccurate channel assessment and the RTS/CTS message exchange becomes prohibitive. Therefore, in this subsection, we will demonstrate a class of protocols which can achieve this *without any coordination* between the nodes.

B. Distributed MAC for Multibeam Systems (DM^{2}S)

In this section, we now describe an Aloha-esque MAC layer protocol which doesn't require any coordination between the nodes.

To send a packet, a node will wait a random amount of time R, then enter transmitting mode. While transmitting, the node will send a packet to *all* of it's neighbors that it can (depending on n and B_{max}).[4] During the waiting time, a node is in receive mode and can successfully receive a packet from any of it's neighbors if it arrives entirely within the receive window (i.e., it does not overlap with the start or end of the interval). After transmitting the packet, the node again waits a random amount of time before sending the next packet. The probability density function and cumulative distribution function are represented as $f_R(r)$ and $F_R(r)$, respectively.

This abstraction of a MAC protocol contains many variations of existing MAC protocols. For example, the well known slotted Aloha can be thought of as having R distributed according to a geometric distribution with discrete unit-length intervals and probability p transmitting in a given slot.

For the scheme where the distribution of the receive window time is arbitrary. We now derive the throughput per link in this scenario:

$$T_{\text{DM}^2\text{S}} = \frac{1}{1+\mathbb{E}[R]} \frac{\mathbb{E}[R]}{1+\mathbb{E}[R]} \mathbb{P}(Y(t) \geq 1). \qquad (2)$$

The first term represents the average time that a link is transmitting. The 2nd term represents the average time the receiver is in receive mode. In the 3rd term, $Y(t)$ represents the forward recurrence time of the receive slot given an arrival at time t. The 3rd term represents the probability that, given a packet arrives while in a receive slot, the receive slot has a remaining length greater than the length of the packet being transmit.

Using well known results from probability theory [23], we have

$$\mathbb{E}[Y(t)] = \frac{\mathbb{E}[R^2]}{2\mathbb{E}[R]} = \frac{\mathbb{E}[R^2]}{2L} \qquad (3)$$

$$\mathbb{P}(Y(t) > 1) = 1 - \frac{1}{\mathbb{E}[R]} \int_0^1 1 - F_R(r) dr \qquad (4)$$

Equation (3) shows that the expected length of the forward recurrance time grows with the second moment of the receive window size. Equation (4), which is valid only for non-lattice distributions suggests that for $\mathbb{P}(Y(t) > L) \to 1$, $F_R(r) \approx 1$ for some small value of r.

Using the insights above, in the following sections, we consider a couple of variations on the distribution of the receive interval duration and quantify the corresponding throughput. Our goal is to find distributions which maximize the throughput. Note that, by ignoring the 3rd term in (2) and attempting to maximize the first two terms, we set $\mathbb{E}[R] = 1$. This means that we would be transmitting for 50% of the time and receiving for 50% of the time for a throughput of 25%, which is the upperbound. However, to maximize the 3rd term such that $\mathbb{P}(Y(t) \geq 1)$, using (4), we must show that,

$$\int_0^1 F_R(r) dr \to 1.$$

[3] A similar problem was considered [22]. However, they use a tree structure to select the set of nodes to transmit and receieve in each slot, providing guarantees of link activations in a superframe. For completeness, we have recreated a similar argument.

[4] Transmissions cannot cause collisions with other links. Hence, if a node is transmitting, it will transmit to as many neighbors as possible.

Intuitively, this means that despite having an expected value at 1, nearly all of the weight of the distribution of R must fall before 1. This observation will be leveraged later on.

Before showing a formal proof characterizing the class of *capacity-achieving* algorithms, we will compute the throughput for commonly used distributions.

Exponential Distributed: In this case, let the receive window length $R \sim$ exponential with mean $E[R]$. Then the throughput can be written as

$$T_{\text{DM}^2\text{S-EXP}} = \frac{L}{E[R]+L}\frac{E[R]}{E[R]+L}\mathrm{e}^{-L/E[R]}.$$

where $\mathbb{E}[R]$ is the expected duration of a receive window. The first term represents the average time that a link is transmitting. The 2nd term represents the average time the receiver is in receive mode. The 3rd term represents the probability that, given a packet arrives while in a receive slot, the receive slot has a length greater *remaining* than the length of the transmit slot. Note that, for the exponential distribution, the memoryless property ensures that the distribution of the remaining time in an interval is equivalent to the distribution of the size of the interval.

This value is maximized at $E[R] = 1 + \sqrt{2}$ for a value of throughput,

$$T_{\text{DM}^2\text{S-EXP}^*} = \frac{C\mathrm{e}^{-1/C}}{(1+C)^2} = 13.68\%$$

where $C = 1 + \sqrt{2}$.

Slotted and Unsynchronized In this scheme, the interarrival time of packets is selected according to gemoetric distribution with discrete intervals of length 1. That is,

$$f_R(r) = \begin{cases} p(1-p)^r & : \forall r = 0,1,2,3\dots \\ 0 & : \text{else} \end{cases}.$$

In this scheme, all nodes divide time into slots where the slot length is the duration of time required to send a packet. In each slot, a node will transmit with probability p. Hence, slotted aloha-like analysis can be utilized to determine the throughput, T.

$$T_{\text{DM}^2\text{S-SLOT}} = \mathbb{P}(\text{Src Tx})\mathbb{P}(\text{Dest Rx for 2 Slots}) \quad (5)$$
$$= p(1-p)^2 \quad (6)$$

Clearly, the throughput is maximized at $p = 1/3$ for a value of

$$T_{\text{DM}^2\text{S-SLOT}^*} = \frac{4}{27} = 14.81\% \quad (7)$$

C. Capacity Achieving Distribution

In the above scenarios, we demonstrated that the exponential and geometric distributions, when optimized, result in a per link throughput of 13.68% and 14.81%, respectively. We hypothesize that the added performance for the slotted distribution over the exponential distribution stems from the added variance of the length of the receive window in the latter case. As alluded to in Section IV-B, the 2nd moment of the receive window is a main factor in the likelihood that a node will interrupt an incoming transmission by beginning the transmit mode. Hence, in this section we consider a MAC scheme which leverages large variances in the receive window size. A similar idea was initially proposed in [24], albeit for omnidirectional networks and no analysis of capacity achieving distributions is given.

Theorem 2: There exists a distribution for the receive window size which results in the random access MAC scheme asymptotically achieving the upperbound on per-link throughput.

Proof: We will prove this theorem through construction of the *Capacity Achieving Distribution (CAD)* which has a receive window size distribution as follows:

$$f_R(r) = \begin{cases} \frac{x-1}{x} & : r = 0 \\ \frac{1}{x} & : r = x \end{cases},$$

where x is referred to as the *backoff time parameter* of the distribution with the constraint that $x > 1$. Recall that 1 is the duration of a packet transmission time. In this distribution there is a (relatively) large probability that the receive window size is of length 0, imply back-to-back packet transmissions. On the other hand, there is a relatively small probability that the receive window is of length x, where x is arbitrary. Hence, we will refer to x as the *delay parameter* as it will represent the amount of time a node must wait for a receive interval to end to send a packet.

Therefore, in this distribution, the mean receive window length is $\mathbb{E}[R] = 1$ and the average time receiving is equal to the average time transmitting. The second moment of the receive window is $\mathbb{E}[R^2] = x$. Therefore, in this distribution, the mean independent of x, but the variance clearly can grow to infinity.

For a given value of x, the probability that the forward recurrence time is greater than the length of the packet at an arbitrary time t is

$$\mathbb{P}(Y(t) > 1) = \frac{x-1}{x}.$$

Therefore,

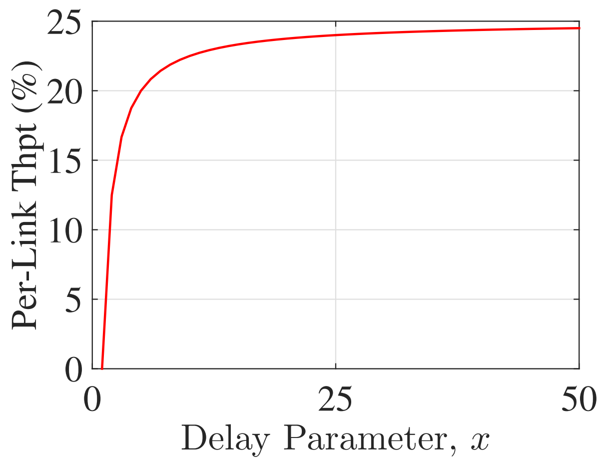
$$\lim_{x\to\infty} T_{\text{DM}^2\text{S-CAD}} = \lim_{x\to\infty} \frac{1}{2}\frac{1}{2}\frac{x-1}{x} = 25\%,$$

and the overall throughput of the system approaches 25%. The relationship between the throughput and the delay parameter, x, is shown in Fig. 3(a). ■

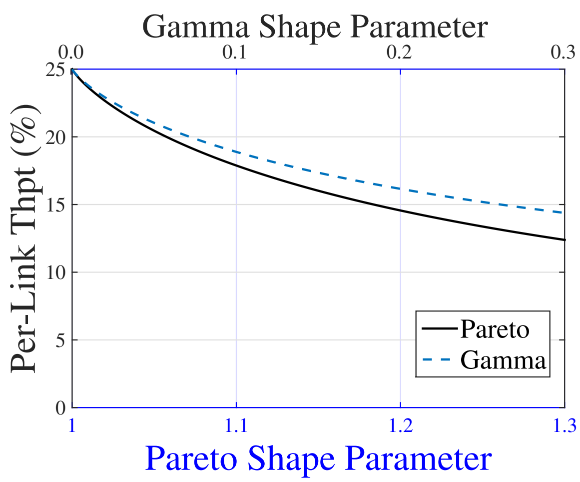
Following this theorem, we now show that infinite variance is a requirement for any capacity-achieving random access MAC scheme.

Lemma 1: For any receive window size distribution with finite variance and unit mean, the DM^{2}S scheme does not achieve the capacity of 25%.

Proof: Clearly, from (2) the throughput can only be 25% when $\mathbb{P}(Y(t) \geq 1) = 1$. For any finite variance distribution, from (4), it is clear that $\mathbb{P}(Y(t) \geq 1) < 1$ and therefore the 25% capacity cannot be achieved. ■



(a)



(b)

Fig. 3: a) The throughput for the capacity achieving distribution approaches the capacity of 25% as the delay parameter $x \to \infty$. b) Random access MAC per-link throughput with R distributed according to the Pareto and Gamma distributions.

D. Infinite Variance Distributions

As demonstrated in the above section, a receive window distribution with finite mean and infinite variance will allow the MAC scheme to approach the upperbound capacity of 25% per-link throughput. In this subsection, we show experimentally that this holds for two well known distributions: the Pareto and the gamma distributions. In Fig. 3(b), we select the scale parameters for both distributions such that the mean receive window length is equal to 1. Then, we vary the shape paraemters. That is, for the pareto distribution the scale parameter $x_m = (\alpha - 1)/\alpha$ where α is the shape parameter. For the gamma distribution, the scale parameter is $1/k$ where k is the shape parameter. As shown in the figure, as we vary the shape parameters towards 0, the corresponding throughput approaches the capacity of 25%.

For the gamma distribution, it is expected that as the shape parameter approaches 0, the variance of the distribution approaches infinity (importantly, with finite mean). This is consistent with the conjecture that it is indeed the variance which is key towards approaching the capacity. However, in the pareto distribution with the shape parameter $1 < \alpha < 2$, the receive window size has infinite variance, independent of the shape parameter. Hence, we find that despite having a distribution with infinite variance, we do not necessarily have a capacity achieving distribution. Instead, the throughput of the random access scheme utilizing the pareto distribution approaches the 25% capacity as the shape parameter approaches 0. We hypothesize that is not the variance, but the 2nd moment growing to infinity which is sufficient condition for a capacity acheiving MAC scheme. Unfortunately, a proof of such a conjecture has thusfar been out of reach and is a subject of future work.

V. Practical Considerations

In the prior Section, we analyzed and demonstrated that the upperbound per-link throughput could be achieved via a random access MAC protocol under idealized physical layer assumptions where there was an infinite queue of packets, no beamwidth ($\theta = 0$), and no power constraints ($B_{\text{max}} = n - 1$). In this section, we relax those assumptions in turn.

First, we consider the scenario where packets arrive according to a poisson arrival process. Then, we consider the case when the the spatial separation between nodes must be greater than θ to transmit or to receive from a neighbor. Last, we consider the per-link throughput in the presence of a power constraint.

A. Stochastic Arrivals

In this subsection, we relax the constraint of all nodes having an infinite buffer of packets to send. Instead, packets arrive according to a stochastic process with arrival rate λ to each node. When a packet arrives, a distinct packet is generated for each of the $n - 1$ neighbors of a node.

This system can then be analyzed as an M/G/1 queue [25] which has a steady-state probability of having a non-empty queue of $\pi_0 = 1 - \lambda/\mu$ where λ/μ is the utilization of the queue based on an arrival rate λ and average service rate μ.

To determine the throughput, we will utilize a similar approach as Section IV. First, we find the percentage of time that a given link is activated by the source. Then, we find the probability that a given packet is successfully received. The MAC scheme is as per the above section.

A node will transmit if it has at least 1 packet in the queue and it is in a transmission window. Hence, it transmits with rate $(1 - \pi_0) * 1/(1 + E[R]) = \rho * 1/2$. A packet is successfully received if the receiving node a) has an empty queue with no new arrivals for a packet duration, b) has an empty queue with an arrival sometime in the next unit time but decides to wait to transmit, or c) has a non-empty queue and is in the receive mode for length at least unit time. These 3 events are independent and the probability of success can be written as:

$$\pi_0 e^{-\lambda} + \pi_0 (1 - e^{-\lambda}) * 1/x + (1 - \pi_0) * \frac{1}{2} \frac{x - 1}{x}.$$

Then the overall throughput is

$$T^{\lambda}_{\mathrm{DM^2S\text{-}CAD}} = \frac{\rho^2}{4}\frac{x-1}{x} + \frac{\rho(1-\rho)}{2}(\mathrm{e}^{-\lambda} + \frac{1-\mathrm{e}^{-\lambda}}{x}). \quad (8)$$

It is easy to see that as $\rho \to 1, x \to \infty$, the queues become backlogged and the throughput approaches the same 25% upperbound throughput from Section IV-A. However, this model allows us to observe the tradeoff for a stochastic arrival process between *throughput* and *delay*.

To compute the delay, we will utilize the well-known *Pollaczek-Khinchin* formula [25] which states that the total delay in system is

$$\text{Delay} = \overline{Y} + \frac{\lambda \overline{Y}^2}{2(1-\rho)},$$

where $\overline{Y}$ and $\overline{Y}^2$ is the average service time and second moment of service time, respectively. The distribution of service time of a packet is:

$$f_Y(y) = \begin{cases} \frac{x-1}{x} & : y = 1 \\ \frac{1}{x} & : y = x+1 \end{cases}.$$

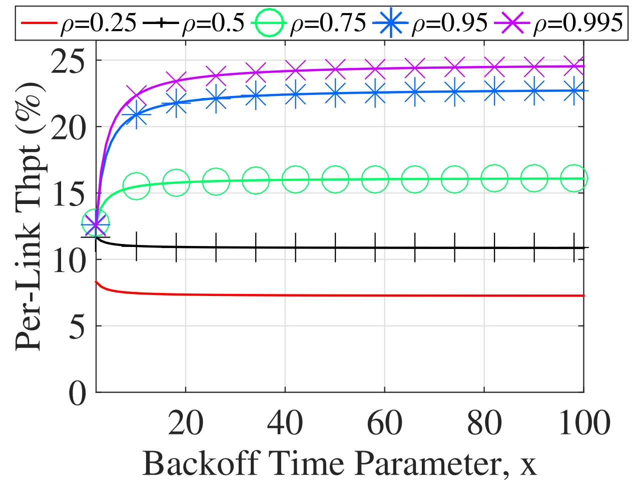
Recall that x is the parameter of the capacity achieving distribution. The service time distribution is simply the delay from the MAC protocol with the added unit-time required to transmit the packet. Then, $\mathbb{E}[\overline{Y}] = \frac{x-1}{x} + \frac{x+1}{x} = 2$ and $\mathbb{E}[\overline{Y}^2] = \frac{x-1}{x} + \frac{(x+1)^2}{x} = 3+x$. The delay can then be written as:

$$\text{Delay} = 2 + \frac{\lambda(x+3)}{2(1-\lambda/2)}. \quad (9)$$

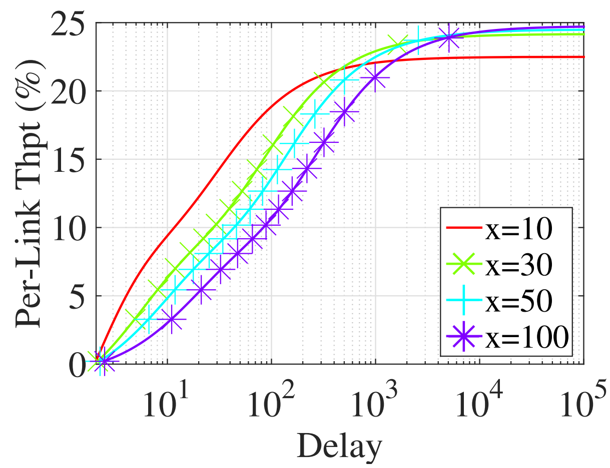
Fig. 4 shows the relationship between throughput (8) and delay (9).

Fig. 4(a) shows the throughput for various utilization rates ρ. For larger values of ρ, the throughput improves with larger backoff times as the system behaves similar to a system with a queue that never empties. Corresponding to a system with an infinte queue of packets, as $\rho \to 1$ and $x \to \infty$, the system approaches the upperbound throughput of 25%. However, for smaller values of ρ, the best scheme is to simply have no backoff time and to immediately transmit a packet when it arrives.

In Fig. 4(b), ρ is varied for various values of x and the corresponding delay D from (9) and throughput from (8) are shown. Again, for smaller values delay, we see that larger values of backoff time result in larger throughput. However, at some threshold it becomes beneficial to increase the backoff time parameter. Also, note that the delay increases linearly in a logarithmic scale with the throughput for small values of delay. There is an elbow in the curve around 10^3 slots at which point the growth to 25% throughput is relatively slow. This implies that a practical scheme may settle for slightly less than optimal throughput for better delay performance



(a)



(b)

Fig. 4: Relationship between per-link throughput ($T_{\mathrm{DM^2S\text{-}CAD}}$) and delay ($D$): a) Link throughput as a function of the backoff time parameter of the CAD for the DM^{2}S scheme and each node has a utilization of ρ, b) Link throughput as a function of the delay (in slots) for various values of the backoff time parameter x.

B. Beamwidth (θ)

From the perspective a receiving node, neighbors must be spatially distributed by at least θ degrees in order to decode the separate streams. Thereby, we now condition on the case where a destination node is receiving from some source node according to the capacity achieving MAC protocol described above. To facilitate this analysis, we will now assume that nodes are uniformly distributed according to a two dimensional spatial poisson process with mean n neighbors per square unit area. Thus far, we have not been able to quantify what the upperbound on per-link throughput would be given the beamdwidth constraint. However, we will now derive the expected per link throughput for the DM^{2}S protocol.

In this case, we focus our analysis on a given link between to a neighbor. The stream can correctly be decoded if no other neighbors within θ degrees is transmitting for duration of a

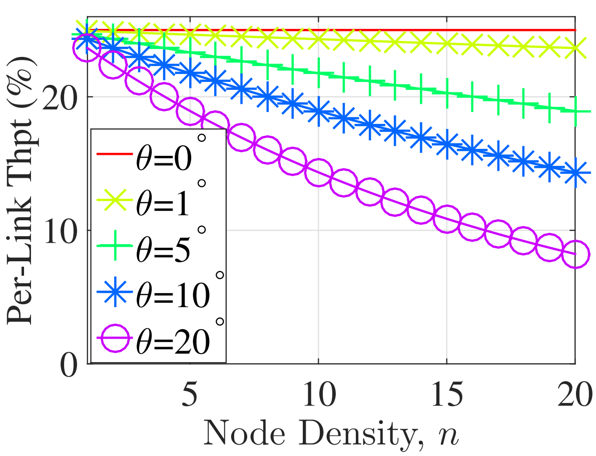


Fig. 5: Relationship between per-link throughput (T^{θ}) and node density n when nodes are distributed according to a spatial poisson process and have a beamdwidth constraint θ.

packet. To determine the expected drop in throughput, we define the set of number of neighbors within θ degrees as a random variable we will denote as Z. For this analysis we will assume that the DM^{2}S-CAD protocol is being used with the backoff time parameter $x = \infty$.

$$\begin{aligned} T^{\theta}_{\text{DM}^2\text{S-CAD}} &= T_{\text{DM}^2\text{S-CAD}}\mathbb{P}(Z \text{ rx'ing for 1 slot}) \\ &= \frac{1}{4}\sum_{z=0}^{\infty}\mathbb{P}(Z \text{ rx'ing } |Z = z)\mathbb{P}(Z = z) \\ &= \frac{1}{4}\sum_{z=0}^{\infty}(\frac{1}{2})^z\frac{(n\theta/360)^z}{z!}\mathrm{e}^{-n\theta/360} \\ &= \frac{1}{4}\mathrm{e}^{-n\theta/360} \end{aligned}$$

This result is quantified in Fig. 5.

C. Power (B_{max})

In this subsection, we assume that there is a power constraint limiting the number of transmit beams that a node can form to B_{max}. Using a similar argument as Section IV-A, we derive the upperbound of the per link throughput as

$$T^{*,B_{\text{max}}} = \begin{cases} B_{\text{max}}(n - B_{\text{max}})/n^2 & : B_{\text{max}} \leq n/2 \\ 25\% & : B_{\text{max}} \geq n/2 \end{cases}.$$

As can be seen, the upperbound throughput with the power constraint is identical to the upperbound without a power constraint if $B_{\text{max}} \geq n/2$. This is because in the upperbound solution, each transmitting node sends to $n/2$ of their neighbors, hence they only need form $n/2$ transmit beams.

For the DM^{2}Sprotocol, we then adapt it such that when transmitting to neighbors, choose the set of B_{max} neighbors to transmit to randomly. Therefore, when transmitting, a transmission beam will be sent to a neighbor with probability

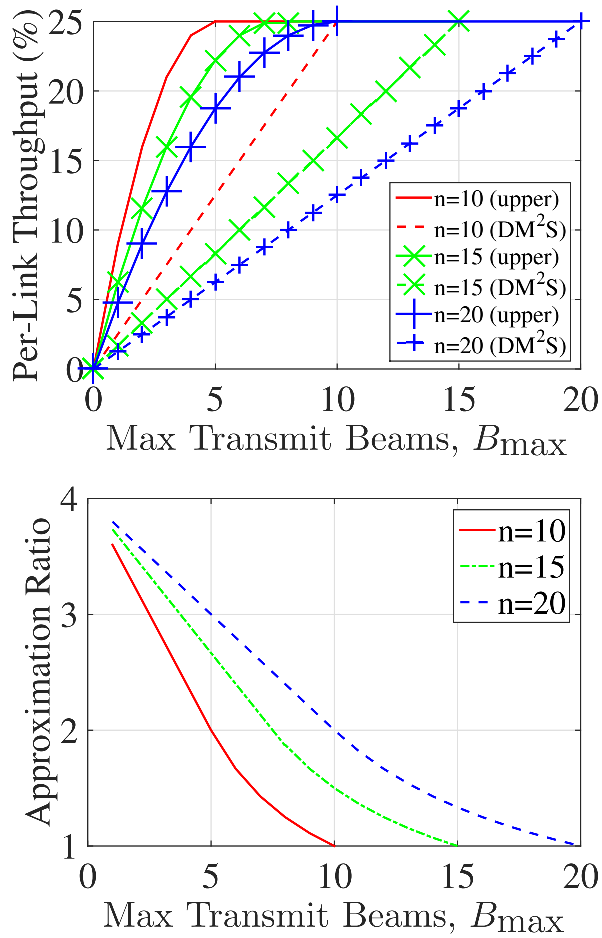


Fig. 6: Relationship between per-link throughput and number of transmit beams B_{max}: a) upperbound per-link throughput ($T^{*,B_{\text{max}}}$) compared to DM^{2}S per-link throughput ($T^{B_{\text{max}}}_{\text{DM}^2\text{S-CAD}}$), b) approximation ratio between $T^{*,B_{\text{max}}}$ and $T^{B_{\text{max}}}_{\text{DM}^2\text{S-CAD}}$.

B_{max}/n and the throughput for the DM^{2}S protocol can then be derived as

$$T^{B_{\text{max}}}_{\text{DM}^2\text{S-CAD}} = T_{\text{DM}^2\text{S-CAD}}B_{\text{max}}/n.$$

The resulting throughput and approximation ratios can be found in Fig. 6. The approximation ratio varies between 4 (at $B_{\text{max}} = 1$) to 2 (at $B_{\text{max}} = n/2$) to 1 (at $B_{\text{max}} = n$).

VI. Conclusions

We consider a communications network of airborne platforms where each node is equipped with digital multi-beamforming antennas. With this technology, a node can form multiple simultaneous transmit or receive beams under the constraint of no-transmit-while-receive. We first present an idealized model of these physical layer capabilities. Using this model, we derive an upper bound on the network capacity assuming an optimal MAC protocol. We then present the Distributed MAC for Multi-beam Systems (DM^{2}S) scheme and show that this random access protocol achieves the performance bound. We assess the resulting delay implications. We

then relax our initial idealistic assumptions and consider the impact of numerous practical considerations including power constraints, beam widths, stochastic arrivals, and latency requirements on the performance of the new random access MAC protocol.

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